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Dynamic response of a poroelastic half space to horizontal buried loading

Bo Jin [a,*], Hua Liu [b]

[a] *Institute of Structure Theory, Tongji University, Shanghai 200092, People's Republic of China*
[b] *College of Mechanical Engineering, Tongji University, Shanghai 200092, People's Republic of China*

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Abstract

This investigation is concerned with the motion of a three-dimensional poroelastic half space produced by a horizontal buried loading. The Green's function for a horizontal point force buried in a poroelastic half space is given as a superposition of the singular solution for the whole space plus a contribution representing relevant effects due to the presence of the free surface. The mathematical approach is based on integral transform techniques. The pressure-solid displacement form of the harmonic equations of motion for a poroelastic solid are developed from the form of the equations originally presented by Biot. The singular solution of point force for the whole space is obtained from thermoelasticity theory according to the analogy between the coupled, dynamic thermoelasticity and the dynamic poroelasticity in the frequency domain. The asymmetric surface contributions in the transformed domain is derived by using the integral transform techniques. The numerical results for a horizontal point loading applied at a finite depth below the surface are presented. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Poroelastic half space; Dynamic response; Horizontal buried loading

1. Introduction

The study of wave propagation in a poroelastic medium is of fundamental importance to several disciplines such as geotechnical engineering, seismology, geophysics and biomechanics. The first theory of propagation of elastic waves in a fluid-saturated porous medium was established by Biot (1956). Biot (1962) also extended his theory to the cases of anisotropic and viscoelastic media. The Green's functions for poroelastic full plane (or full space) were presented by Bonnet (1987), Manolis and Beskos (1989), Dominguez (1991) and Cheng et al. (1991). Two-dimensional dynamic Green's functions of homogeneous poroelastic half-plane were derived by Senjuntichai and Rajapakse (1994). The responses of poroelastic half space due to vertical loadings acting on the surface and at a finite depth below the surface were considered

* Corresponding author. Tel.: +86-21-6501-7470.
E-mail address: jinliuyk@online.sh.cn (Bo Jin).

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by Halpern and Christiano (1986a) and Philippacopoulos (1988, 1997). The interaction of a loaded plate with a saturated, and layered saturated poroelastic half space have been studied by Halplern and Christiano (1986b), Philippacopoulos (1989), Bougacha et al. (1993), Rajapakse and Senjuntichai (1995), Jin (1999) and Jin and Liu (1999). Dynamic axial load transfer from elastic bar to poroelastic medium has also been discussed (Zeng and Rajapakse, 1999). A comprehensive review on the dynamic analysis of poroelastic media with boundary element methods has been presented by Beskos (1997). It is worth mentioning that, although many problems involving dynamic response of poroelastic medium have been studied, the dynamic responses of a three-dimensional poroelastic half space to lateral buried dynamic excitations are not reported in the literature. Solutions for internal, lateral excitations can be used in the study of dynamic response of embedded foundations under lateral loading.

The main objective of this paper is to present a solution corresponding to lateral dynamic loading applied at a finite depth below the surface of a poroelastic half space. It was demonstrated by Bonnet (1987) and Cheng et al. (1991) that there exists a complete analogy between the coupled, dynamic thermoelasticity and the dynamic poroelasticity in the frequency domain. By eliminating the fluid displacement relative to matrix Bonnet reduced Biot's wave equations to the partial differential equations expressed in terms of the displacements of the solid matrix and the pore-fluid pressure. Bonnet (1987) and Cheng et al. (1991) obtained poroelastic Green's functions for a full plane and a full space from the solutions given by Kupradze et al. (1979) and Nowacki (1975), respectively, for thermoelasticity. However solutions involving poroelastic half plane (or half space) cannot be derived by using analogy between the thermoelasticity and the poroelasticity since the thermoelastic solution for a loaded half space are not available in the literature. In this paper a general solution involving poroelastic half space for the asymmetric problem is obtained by the method of integral transform, which are presented in Section 4. By using the method of superposition (Philippacopoulos, 1997), the required result can be derived, the details of which are furnished in Section 5. In Section 6, illustrative numerical results are presented.

2. Governing equations

At this stage, it is convenient to non-dimensionalize all quantities with respect to length and stress by selecting a length of reference a as a unit of length and the shear modulus of half space as a unit of stress. Making use of the method proposed by Bonnet (1987) we can reduce Biot's wave equations to the following dimensionless equations:

$$u_{i,jj} + (\lambda^* + 1)u_{j,ji} + a_0^2(1 - \rho^*\vartheta)u_i - (\alpha - \vartheta)p_{,i} + F_i = 0 \tag{1}$$

$$p_{,ii} + \frac{\rho^* a_0^2}{M^*\vartheta}p + \frac{\rho^* a_0^2(\alpha - \vartheta)}{\vartheta}e = 0 \tag{2}$$

in which $F_i = \delta_{1i}\delta(x_1)\delta(x_2)\delta(x_3 - h)$ where h is the depth of the horizontal buried load; $e = u_{i,i}$ is solid strain and u_i $(i = 1, 2, 3)$ are the dimensionless solid displacements; p is the dimensionless pore-fluid pressure; α, $M^* = M/\mu$, $\lambda^* = \lambda/\mu$, $\rho^* = \rho_f/\rho$, $m^* = m/\rho$ and $b^* = ab/(\rho\mu)^{1/2}$ are the dimensionless material parameters, where λ and μ are Lame constants; α and M are Biot's parameters accounting for compressibility of the two-phased material; ρ and ρ_f are mass densities of the bulk material and the pore fluid, respectively; m is a density-like parameter that depends on ρ_f and the geometry of the pores; b is a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid. The parameter b is equal to the ratio between the fluid viscosity and the intrinsic permeability of the medium. If internal friction is neglected then $b = 0$. $\vartheta = \rho^* a_0^2/(m^* a_0^2 - ib^* a_0)$ is a dimensionless parameter. $a_0 = \omega a(\rho/\mu)^{1/2}$ is a dimensionless frequency where ω is the frequency of the motion. For brevity, the time factor of e^{ia_0t} where t

is a dimensionless time has been omitted from Eqs. (1) and (2) and also from the sequel. Green's function of a poroelastic half space due to a buried horizontal point force can be obtain by using the method of superposition. The problem is the superposition of the following two problems.

3. The solution of full space due to horizontal point force

According to Cheng et al. (1991), the solution of a poroelastic full space due to horizontal point applied at $(0,0,h)$ can be expressed in cylindrical coordinates (r,θ,z) as follows:

$$u_r = \frac{\cos\theta}{4\pi\mu a^2 S^2}\left[\frac{\partial^2}{\partial r^2}\left(-\delta_1\frac{e^{-iL_1R}}{R}+\delta_2\frac{e^{-iL_2R}}{R}+\frac{e^{-iSR}}{R}\right)+S^2\frac{e^{-iSR}}{R}\right] \tag{3}$$

$$u_0 = \frac{-\sin\theta}{4\pi\mu a^2 S^2}\left[\frac{\partial}{r\partial r}\left(-\delta_1\frac{e^{-iL_1R}}{R}+\delta_2\frac{e^{-iL_2R}}{R}+\frac{e^{-iSR}}{R}\right)+S^2\frac{e^{-iSR}}{R}\right] \tag{4}$$

$$u_z = \frac{\cos\theta}{4\pi\mu a^2 S^2}\frac{\partial^2}{\partial r\partial z}\left(-\delta_1\frac{e^{-iL_1R}}{R}+\delta_2\frac{e^{-iL_2R}}{R}+\frac{e^{-iSR}}{R}\right) \tag{5}$$

$$p = \frac{\delta_3\cos\theta}{4\pi\mu a^2}\frac{\partial}{\partial r}\left(\frac{e^{-iL_1R}}{R}-\frac{e^{-iL_2R}}{R}\right) \tag{6}$$

where

$$L_1^2 = \frac{\beta_1+\sqrt{\beta_1^2-4\beta_2}}{2} \tag{7}$$

$$L_2^2 = \frac{\beta_1-\sqrt{\beta_1^2-4\beta_2}}{2} \tag{8}$$

$$S^2 = (1-\rho^*\vartheta)a_0^2 \tag{9}$$

$$\beta_1 = \frac{(m^*a_0^2-ib^*a_0)(\lambda^*+\alpha^2M^*+2)+M^*a_0^2-2\alpha M^*\rho^*a_0^2}{(\lambda^*+2)M^*} \tag{10}$$

$$\beta_2 = \frac{(m^*a_0^2-ib^*a_0)a_0^2-(\rho^*)^2a_0^4}{(\lambda^*+2)M^*} \tag{11}$$

$$R = \sqrt{r^2+(z-h)^2} \tag{12}$$

$$\delta_i = \frac{a_0^2(1-\vartheta\rho^*)(\vartheta M^*L_i^2-\rho^*a_0^2)}{(\lambda^*+2)(L_1^2-L_2^2)M^*\vartheta L_i^2}\quad (i=1,2) \tag{13}$$

$$\delta_3 = \frac{\rho^*a_0^2(\alpha-\vartheta)}{(\lambda^*+2)(L_1^2-L_2^2)\vartheta} \tag{14}$$

It is convenient to consider the displacements, stresses and pore-fluid pressure that can be expressed as follows:

$$\left\{\begin{array}{c} u_r(r,\theta,z) \\ u_z(r,\theta,z) \\ \tau_{rz}(r,\theta,z) \\ \sigma_z(r,\theta,z) \\ p(r,\theta,z) \end{array}\right\} = \left\{\begin{array}{c} u_{r1}(r,z) \\ u_{z1}(r,z) \\ \tau_{rz1}(r,z) \\ \sigma_{z1}(r,z) \\ p_1(r,z) \end{array}\right\}\cos\theta, \qquad \left\{\begin{array}{c} u_\theta(r,\theta,z) \\ \tau_{\theta z}(r,\theta,z) \end{array}\right\} = \left\{\begin{array}{c} u_{\theta 1}(r,z) \\ \tau_{\theta z1}(r,z) \end{array}\right\}\sin\theta \tag{15}$$

Defining

$$U_1 = u_{r1} + u_{\theta 1}, \quad V_1 = u_{r1} - u_{\theta 1}, \quad X_1 = \tau_{rz1} - \tau_{\theta z1}, \quad Y_1 = \tau_{rz1} + \tau_{\theta z1} \tag{16}$$

and considering the Sommerfeld integral

$$\frac{\mathrm{e}^{-\mathrm{i}k_\alpha R}}{R} = \int_0^\infty \xi \frac{\mathrm{e}^{-\gamma_\alpha |z-h|}}{\gamma_\alpha} J_0(\xi r)\,\mathrm{d}\xi \tag{17}$$

where

$$\gamma_\alpha = \sqrt{\xi^2 - k_\alpha^2} \tag{18}$$

and the following identity

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} \pm \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) J_0(\xi r) = \mp \xi^2 J_{1\mp 1}(\xi r) \tag{19}$$

the displacements, stresses and pore pressure for full space in the transformed domain can be obtained, which are given by Eqs. (A.1)–(A.10) in Appendix A.

4. The solution of half space due to surface loadings

The homogeneous solution of Eqs. (1) and (2) in the transformed domain for asymmetric problem can be obtained by using the method of integral transform. Putting $F_i = 0$ in Eq. (1), and after some manipulations we obtain the transformed solutions for half space as follows:

$$\overline{U}_1(\xi,z) = \frac{2}{\xi}\left(g_1 - \frac{3\xi^2 a_1}{2}\right) A_1 \mathrm{e}^{-\gamma_1 z} + \frac{2}{\xi}\left(g_2 - \frac{3\xi^2 a_2}{2}\right) B_1 \mathrm{e}^{-\gamma_2 z} + \left(\frac{2}{\xi} C_1 \gamma_3 + D_1\right)\mathrm{e}^{-\gamma_3 z} \tag{20}$$

$$\overline{V}_1(\xi,z) = -\xi a_1 A_1 \mathrm{e}^{-\gamma_1 z} - \xi a_2 B_1 \mathrm{e}^{-\gamma_2 z} + D_1 \mathrm{e}^{-\gamma_3 z} \tag{21}$$

$$\overline{u}_{z1}(\xi,z) = \gamma_1 a_1 A_1 \mathrm{e}^{-\gamma_1 z} + \gamma_2 a_2 B_1 \mathrm{e}^{-\gamma_2 z} + C_1 \mathrm{e}^{-\gamma_3 z} \tag{22}$$

$$\overline{p}_1(\xi,z) = A_1 \mathrm{e}^{-\gamma_1 z} + B_1 \mathrm{e}^{-\gamma_2 z} \tag{23}$$

Making use of the transformed form of the constitutive relations, we obtain:

$$\overline{X}_1(\xi,z) = 2\xi\gamma_1 a_1 A_1 \mathrm{e}^{-\gamma_1 z} + 2\xi\gamma_2 a_2 B_1 \mathrm{e}^{-\gamma_2 z} + (C_1\xi - \gamma_3 D_1)\mathrm{e}^{-\gamma_3 z} \tag{24}$$

$$\overline{Y}_1(\xi,z) = -\frac{2\gamma_1}{\xi}(\chi_1+\gamma_1^2 a_1)A_1 \mathrm{e}^{-\gamma_1 z} - \frac{2\gamma_2}{\xi}(\chi_2+\gamma_2^2 a_2)B_1 \mathrm{e}^{-\gamma_2 z} - \left[\left(\xi+\frac{2}{\xi}\gamma_3^2\right)C_1+\gamma_3 D_1\right]\mathrm{e}^{-\gamma_3 z} \tag{25}$$

$$\overline{\sigma}_{z1}(\xi,z) = g_3 A_1 \mathrm{e}^{-\gamma_1 z} + g_4 B_1 \mathrm{e}^{-\gamma_2 z} - 2\gamma_3 C_1 \mathrm{e}^{-\gamma_3 z} \tag{26}$$

where

$$\chi_i = \frac{\vartheta M^* L_i^2 - \rho^* a_0^2}{\rho^* a_0^2(\alpha-\vartheta)M^*}, \quad i=1,2 \tag{27}$$

$$a_i = \frac{\lambda^*\chi_i + \chi_i - \alpha + \vartheta}{S^2 - L_i^2}, \quad i=1,2 \tag{28}$$

$$g_1 = \chi_1 + (\gamma_1^2+\xi^2)a_1, \quad g_2 = \chi_2 + (\gamma_2^2+\xi^2)a_2 \tag{29}$$

$$g_3 = \lambda^*\chi_1 - 2\gamma_1^2 a_1 - \alpha, \quad g_4 = \lambda^*\chi_2 - 2\gamma_2^2 a_2 - \alpha \tag{30}$$

Next, functions $A_1(\xi)$, $B_1(\xi)$, $C_1(\xi)$ and $D_1(\xi)$ are determined for four surface conditions. Solving Eqs. (23)–(26) yields:

$$\begin{Bmatrix} A_1(\xi) \\ B_1(\xi) \\ C_1(\xi) \\ D_1(\xi) \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \begin{Bmatrix} \overline{X}_1(\xi,0) \\ \overline{Y}_1(\xi,0) \\ \overline{\sigma}_{z1}(\xi,0) \\ \overline{p}_1(\xi,0) \end{Bmatrix} \tag{31}$$

where f_{ij} $(i=1,4,\ j=1,4)$ are given by Eqs. (A.11)–(A.22) in Appendix A.

5. Using the method of superposition to obtain the solution of a poroelastic half space due to a buried horizontal point force

The solution of a poroelastic half space due to a buried horizontal point force can be now obtained as follows: setting $z=0$ in Eqs. (A.4), (A.8)–(A.10), take opposite signs of the result and substitute it into Eq. (31), then substituting the resulting four sets $A_1(\xi)$, $B_1(\xi)$, $C_1(\xi)$ and $D_1(\xi)$ into Eqs. (20)–(23). Using the superposition principle we obtain the displacements and pore pressure in the transformed domain, then taking inverse Hankel transforms the explicit solutions can expressed as follows:

$$u_{r1}(r,z) = \frac{1}{2}\left[\int_0^\infty \xi\overline{U}_1(\xi,z)J_2(\xi r)\,\mathrm{d}\xi + \int_0^\infty \xi\overline{V}_1(\xi,z)J_0(\xi r)\,\mathrm{d}\xi\right] \tag{32}$$

$$u_{\theta 1}(r,z) = \frac{1}{2}\left[\int_0^\infty \xi\overline{U}_1(\xi,z)J_2(\xi r)\,\mathrm{d}\xi - \int_0^\infty \xi\overline{V}_1(\xi,z)J_0(\xi r)\,\mathrm{d}\xi\right] \tag{33}$$

$$u_{z1}(r,z) = \int_0^\infty \xi\overline{u}_{z1}(\xi,z)J_1(\xi r)\,\mathrm{d}\xi \tag{34}$$

$$p_1(r,z) = \int_0^\infty \xi\overline{p}_1(\xi,z)J_1(\xi r)\,\mathrm{d}\xi \tag{35}$$

where $\overline{U}_1(\xi,z)$, $\overline{V}_1(\xi,z)$, $\overline{u}_{z1}(\xi,z)$ and $\overline{p}_1(\xi,z)$ are given by Eqs. (A.23)–(A.26) in Appendix A.

The method of superposition, which uses the fundamental solution of full space with superposition to derive half space solutions, requires the evaluation of fewer integration constants. To quantify this difference, recall that for wave propagation problems in poroelastic media, the number of integration constants increases from the elastic case, since, in a source-free infinite poroelastic space, the propagation involves three waves rather than two. Therefore, following the approach proposed by Pak (1987), which treats half space as a two-domain problem, the solution of a 12×12 system is required to compute the response of the poroelastic half space, in contrast to the method of superposition, which requires the solution of a four by four system. However, an important advantage of the approach which treats half space as a two-domain problem is that it does not require prior knowledge of the corresponding Green's function for the full space.

6. Numerical results

The solutions for displacements and pore pressure are given by Eqs. (32)–(35). It is found from Eqs. (A.23)–(A.26) in Appendix A that each integrand in Eqs. (32)–(35) consists of 12 terms: the first three represent source terms and the remaining nine represent surface terms. Since source terms can be evaluated in closed forms (Eqs. (A.1)–(A.4)), all that is required is evaluation of the integral representation of surface terms. However, the integral representation of surface contribution cannot be evaluated analytically. In view of the complexity of the integrands, it is natural to employ a suitable numerical quadrature scheme to evaluate the integrals. The singularities of the integrands need to be investigated before the establishment of a numerical integration procedure. The important singularities of the integrands are the branch points of the radicals γ_i $(i = 1, 2, 3)$ as defined by Eqs. (A.5) and (A.6) and poles of the function Δ defined in Eq. (A.22). The branch points are given by L_1, L_2, and S, while poles are given by the roots of the following equation:

$$\Delta = (\gamma_3^2 + \xi^2)(g_4 - g_3) - 2\gamma_3(\gamma_1 g_1 - \gamma_2 g_2) = 0$$

which is the Rayleigh equation for a poroelastic half space governing the propagation of the surface waves. It is noted that the surface wave for a poroelastic medium is dispersive and dissipative if internal friction exists (i.e. $b \neq 0$). Generally, these branch points and poles are all complex valued. However, their locations can be on the real axis if the viscous coupling between the solid matrix and the pore fluid is neglected ($b = 0$). In this paper, the dissipative nature of the half space is incorporated (i.e. $b \neq 0$) therefore the real ξ-axis is free from any singularities.

Since the $\mathrm{Re}(\xi)$-axis is free from singularities, subsequent numerical applications will be based on integration along this axis. The numerical solutions can be computed by using direct numerical quadrature scheme such as the extended trapezoidal rule or Guass quadrature. In this paper, the numerical solutions are obtained by using the extended trapezoidal formula with a sampling interval of $\Delta\xi$. It is noted that when the path of integration is in the neighborhood of the pole, the integrand become nearly singular and a very small integration interval has to be employed. Therefore, $\Delta\xi = 0.002$ for $|\xi - \mathrm{Re}(\xi_R)| < 0.2$ where ξ_R is the pole given by the Rayleigh equation for a poroelastic half space and $\Delta\xi = 0.05$ where ξ is outside that region.

In the numerical study the displacements and pore pressure of poroelastic half space due to horizontal buried unit point force applied at (0, 0, $h = 1$) and in the $\theta = 0$ direction are considered. The dimensionless parameters of the poroelastic materials are: $\lambda^* = 1.5$, $\rho^* = 0.53$, $m^* = 1.1$, $M^* = 5$, $\alpha = 0.95$. In addition, $b^* = 0.5$, 5 and 50. The larger the permeability, the smaller the b^* is. In the figures asterisk, open circle and filled circle represent $b^* = 0.5$, 5 and 50.

Figs. 1 and 2 show the radial displacement u_r $(r = 1, \theta = 0, z)$ varying with z for three different poroelastic materials. Solutions are presented for two different frequencies ($a_0 = 1$ and 3). It is evident from these solutions that the response of the half space depends very significantly on the frequency of the excitation of the loading. Both real and imaginary parts of the displacements shown in Figs. 1 and 2 vary rapidly with the distance and become more oscillatory as the frequency of excitation increases. Comparison of solutions presented in Figs. 1 and 2 also indicate that the poroelastic properties of the medium (b^*) has a significant influence on the response.

Figs. 3 and 4 show the vertical displacement $u_z(1, 0, z)$ varying with z due to horizontal buried point force. Solutions are given for two frequencies of excitations. It is noted that the vertical displacement is much less than the radial displacement observed earlier in Figs. 1 and 2. Figs. 3 and 4 indicate that at low

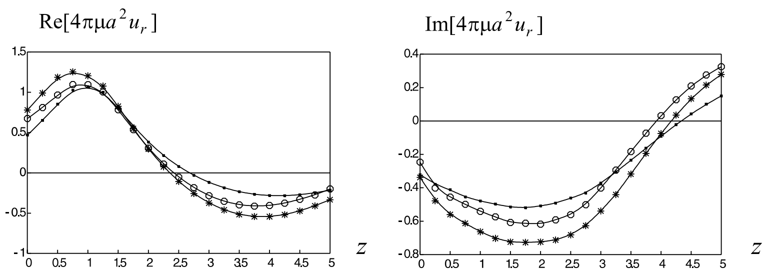

Fig. 1. Radial displacement $u_r(1, 0, z)$ for the case of $h = 1$ at $a_0 = 1$.

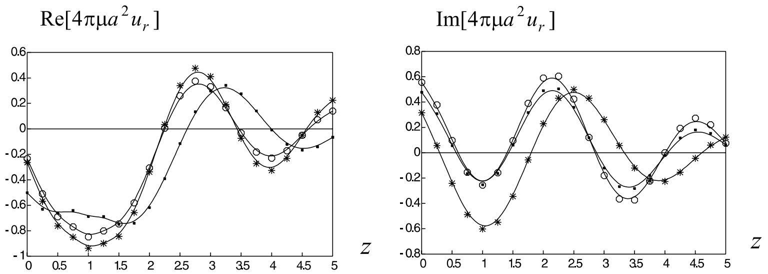

Fig. 2. Radial displacement $u_r(1, 0, z)$ for the case of $h = 1$ at $a_0 = 3$.

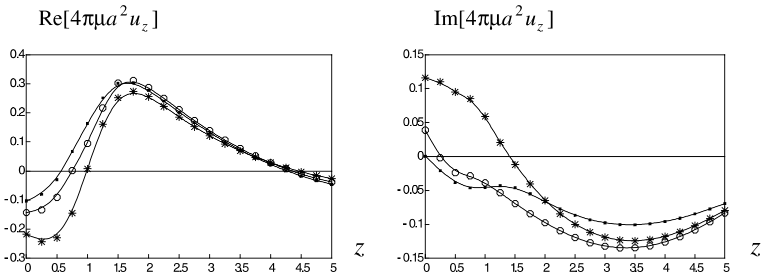

Fig. 3. Vertical displacement $u_z(1, 0, z)$ for the case of $h = 1$ at $a_0 = 1$.

frequencies ($a_0 = 1$) the displacements vary rapidly with the depth whereas at high frequencies ($a_0 = 3$) the variations become more oscillatory. In general, the influence of poroelastic properties of the medium on the solutions shown in Figs. 3 and 4 is similar to that observed earlier for the radial displacement $u_r(1, 0, z)$. It is also found that the real part of the vertical displacement is larger at low frequencies than that at high frequencies.

Figs. 5 and 6 show the profiles of pore pressure due to horizontal buried point force for three different poroelastic materials. The pore pressure $p(1, 0, z)$, depends significantly on the frequency and poroelastic material properties. The pore pressure is larger at high frequencies than that at low frequencies. The magnitude of pore pressure is found to increase with increasing b^*, which is consistent with the fact that a higher b^* means a more impermeable medium.

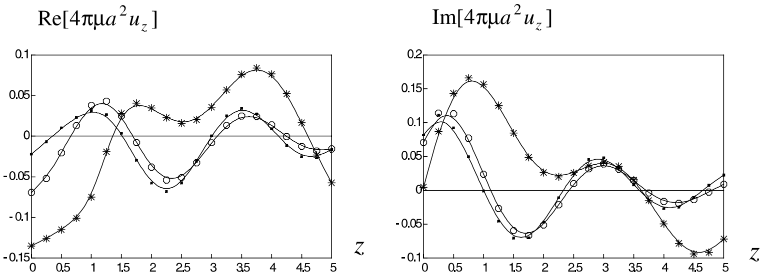

Fig. 4. Vertical displacement $u_z(1, 0, z)$ for the case of $h = 1$ at $a_0 = 3$.

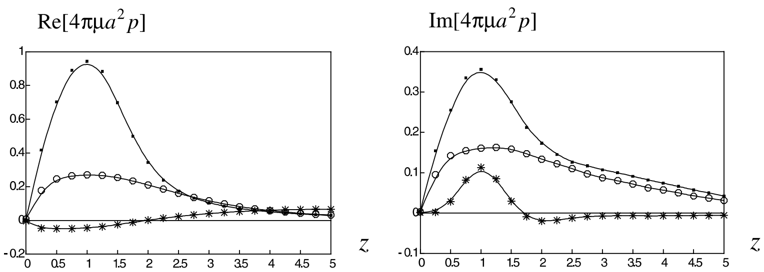

Fig. 5. Pore pressure $p(1, 0, z)$ for the case of $h = 1$ at $a_0 = 1$.

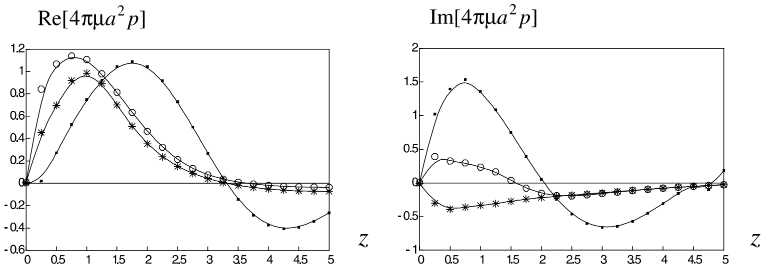

Fig. 6. Pore pressure $p(1, 0, z)$ for the case of $h = 1$ at $a_0 = 3$.

7. Conclusions

In this paper, the response of a three-dimensional poroelastic half space under the action of a time-harmonic, horizontal, buried source is derived analytically by the method of superposition. The solution is given as a superposition of infinite space Green's functions and surface contributions. The surface contributions in the transformed domain for asymmetric conditions is obtained by the method of integral transform. Numerical results presented in the paper indicated that the permeability of poroelastic medium has important influences on the horizontal dynamic responses of a poroelastic half space. Numerical results also show a strong dependence of dynamic response on the frequency of excitation. The solution presented here can be used to study the dynamic response of foundation in a poroelastic half space under lateral loading. With the aid of Fourier or Laplace transforms, the method can be naturally extended to the treatment of transient problems.

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Appendix A

The displacements and pore pressure for full space in the transformed domain are given by

$$\overline{U}_{1\infty} = \frac{1}{4\pi\mu a^2 S^2}\left(\frac{-\delta_1\xi^2 e^{-\gamma_1|z-h|}}{\gamma_1} + \frac{\delta_2\xi^2 e^{-\gamma_2|z-h|}}{\gamma_2} + \frac{\xi^2 e^{-\gamma_3|z-h|}}{\gamma_3}\right) \tag{A.1}$$

$$\overline{V}_{1\infty} = \frac{1}{4\pi\mu a^2 S^2}\left(\frac{\delta_1\xi^2 e^{-\gamma_1|z-h|}}{\gamma_1} - \frac{\delta_2\xi^2 e^{-\gamma_2|z-h|}}{\gamma_2} - \frac{\xi^2 e^{-\gamma_3|z-h|}}{\gamma_3} + \frac{2S^2 e^{-\gamma_3|z-h|}}{\gamma_3}\right) \tag{A.2}$$

$$\overline{u}_{z1\infty} = \frac{\mathrm{sgn}(h-z)}{4\pi\mu a^2 S^2}\left(\delta_1\xi e^{-\gamma_1|z-h|} - \delta_2\xi e^{-\gamma_2|z-h|} - \xi e^{-\gamma_3|z-h|}\right) \tag{A.3}$$

$$\overline{p}_{1\infty} = \frac{\delta_3}{4\pi\mu a^2}\left(\frac{-\xi e^{-\gamma_1|z-h|}}{\gamma_1} + \frac{\xi e^{-\gamma_2|z-h|}}{\gamma_2}\right) \tag{A.4}$$

where

$$\gamma_i = \sqrt{\xi^2 - L_i^2}, \quad i = 1, 2 \tag{A.5}$$

$$\gamma_3 = \sqrt{\xi^2 - S^2} \tag{A.6}$$

$$\mathrm{sgn}(h-z) = \begin{cases} +1, & h > z \\ -1, & h < z \end{cases} \tag{A.7}$$

Note that the radicals γ_i $(i = 1, 2, 3)$ are selected such that $\mathrm{Re}(\gamma_i) \geqslant 0$. Stresses can be expressed as:

$$\overline{X}_{1\infty} = \frac{\mathrm{sgn}(h-z)}{2\pi\mu a^2 S^2}\left(\delta_1\xi^2 e^{-\gamma_1|z-h|} - \delta_2\xi^2 e^{-\gamma_2|z-h|} - \gamma_3^2 e^{-\gamma_3|z-h|}\right) \tag{A.8}$$

$$\overline{Y}_{1\infty} = \frac{\mathrm{sgn}(h-z)}{2\pi\mu a^2 S^2}\left(-\delta_1\xi^2 e^{-\gamma_1|z-h|} + \delta_2\xi^2 e^{-\gamma_2|z-h|} + \xi^2 e^{-\gamma_3|z-h|}\right) \tag{A.9}$$

$$\overline{\sigma}_{z1\infty} = \frac{1}{4\pi\mu a^2 S^2}\left[\frac{\xi(2\delta_1\gamma_1^2 - \lambda^*\delta_1 L_1^2 + \alpha S^2\delta_3)}{\gamma_1} e^{-\gamma_1|z-h|} - \frac{\xi(2\delta_2\gamma_2^2 - \lambda^*\delta_2 L_2^2 + \alpha S^2\delta_3)}{\gamma_2} e^{-\gamma_2|z-h|} - 2\xi\gamma_3 e^{-\gamma_3|z-h|}\right] \tag{A.10}$$

The functions f_{ij} $(i = 1,4,\ j = 1,4)$ are defined as follows:

$$-f_{11} = f_{12} = f_{21} = -f_{22} = \frac{\gamma_3\xi}{\Delta} \tag{A.11}$$

$$-f_{13} = f_{23} = \frac{\gamma_3^2 + \xi^2}{\Delta} \tag{A.12}$$

$$f_{14} = \frac{-2\gamma_2\gamma_3 g_2 - (\xi^2 + \gamma_3^2)g_4}{\Delta} \tag{A.13}$$

$$f_{24} = \frac{2\gamma_1\gamma_3 g_1 + (\xi^2 + \gamma_3^2)g_3}{\Delta} \tag{A.14}$$

$$-f_{31} = f_{32} = \frac{\xi(g_3 - g_4)}{2\Delta} \tag{A.15}$$

$$f_{33} = \frac{\gamma_1 g_1 - \gamma_2 g_2}{\Delta} \tag{A.16}$$

$$f_{34} = \frac{\gamma_2 g_2 g_3 - \gamma_1 g_1 g_4}{\Delta} \tag{A.17}$$

$$f_{41} = \frac{4\xi^2\gamma_3(\gamma_2 a_2 - \gamma_1 a_1) + 4\gamma_3(\gamma_1 g_1 - \gamma_2 g_2) + (2\gamma_3^2 + \xi^2)(g_3 - g_4)}{2\gamma_3\Delta} \tag{A.18}$$

$$f_{42} = \frac{\xi^2(g_3 - g_4) - 4\xi^2\gamma_3(\gamma_2 a_2 - \gamma_1 a_1)}{2\gamma_3\Delta} \tag{A.19}$$

$$f_{43} = \frac{2\xi(\gamma_1 g_1 - \gamma_2 g_2) + 4\xi(\gamma_2 a_2 - \gamma_1 a_1)(\gamma_3^2 + \xi^2)}{2\gamma_3\Delta} \tag{A.20}$$

$$f_{44} = \frac{4\xi\gamma_1\gamma_2\gamma_3(a_1 g_2 - a_2 g_1) + 2\xi(\xi^2 + \gamma_3^2)(\gamma_1 a_1 g_4 - \gamma_2 a_2 g_3) + \xi(\gamma_2 g_2 g_3 - \gamma_1 g_2 g_4)}{\gamma_3\Delta} \tag{A.21}$$

$$\Delta = (\gamma_3^2 + \xi^2)(g_4 - g_3) - 2\gamma_3(\gamma_1 g_1 - \gamma_2 g_2) \tag{A.22}$$

The transformed displacements and pore pressure for half space due to horizontal buried point force are expressed as follows:

$$\begin{aligned}\overline{U}_1(\xi,z) = \overline{U}_{1\infty}(\xi,z) + \frac{1}{4\pi\mu a^2 S^2}\Bigg\{ &\frac{2}{\xi}\left(g_1 - \frac{3\xi^2 a_1}{2}\right)t_{11}\mathrm{e}^{-\gamma_1(z+h)} + \frac{2}{\xi}\left(g_2 - \frac{3\xi^2 a_2}{2}\right)t_{22}\mathrm{e}^{-\gamma_2(z+h)} \\ &+ \left(\frac{2}{\xi}\gamma_3 t_{33} + t_{43}\right)\mathrm{e}^{-\gamma_3(z+h)} + \frac{2}{\xi}\left(g_1 - \frac{3\xi^2 a_1}{2}\right)t_{12}\mathrm{e}^{-(\gamma_1 z+\gamma_2 h)} \\ &+ \frac{2}{\xi}\left(g_1 - \frac{3\xi^2 a_1}{2}\right)t_{13}\mathrm{e}^{-(\gamma_1 z+\gamma_3 h)} + \frac{2}{\xi}\left(g_2 - \frac{3\xi^2 a_2}{2}\right)t_{21}\mathrm{e}^{-(\gamma_2 z+\gamma_1 h)} + \frac{2}{\xi}\left(g_2 - \frac{3\xi^2 a_2}{2}\right)t_{23}\mathrm{e}^{-(\gamma_2 z+\gamma_3 h)} \\ &+ \left(\frac{2}{\xi}\gamma_3 t_{31} + t_{41}\right)\mathrm{e}^{-(\gamma_3 z+\gamma_1 h)} + \left(\frac{2}{\xi}\gamma_3 t_{32} + t_{42}\right)\mathrm{e}^{-(\gamma_3 z+\gamma_2 h)}\Bigg\} \end{aligned} \tag{A.23}$$

$$\begin{aligned}\overline{V}_1(\xi,z) = \overline{V}_{1\infty}(\xi,z) + \frac{1}{4\pi\mu a^2 S^2}\big\{ &-\xi a_1 t_{11}\mathrm{e}^{-\gamma_1(z+h)} - \xi a_2 t_{22}\mathrm{e}^{-\gamma_2(z+h)} + t_{43}\mathrm{e}^{-\gamma_3(z+h)} \\ &- \xi a_1 t_{12}\mathrm{e}^{-(\gamma_1 z+\gamma_2 h)} - \xi a_1 t_{13}\mathrm{e}^{-(\gamma_1 z+\gamma_3 h)} - \xi a_2 t_{21}\mathrm{e}^{-(\gamma_2 z+\gamma_1 h)} - \xi a_2 t_{23}\mathrm{e}^{-(\gamma_2 z+\gamma_3 h)} \\ &+ t_{41}\mathrm{e}^{-(\gamma_3 z+\gamma_1 h)} + t_{42}\mathrm{e}^{-(\gamma_3 h+\gamma_2 h)}\big\} \end{aligned} \tag{A.24}$$

$$\begin{aligned}\overline{u}_{z1}(\xi,z) = \overline{u}_{z1\infty}(\xi,z) + \frac{1}{4\pi\mu a^2 S^2}\big\{ &\gamma_1 a_1 t_{11}\mathrm{e}^{-\gamma_1(z+h)} + \gamma_2 a_2 t_{22}\mathrm{e}^{-\gamma_2(z+h)} + t_{33}\mathrm{e}^{-\gamma_3(z+h)} \\ &+ \gamma_1 a_1 t_{12}\mathrm{e}^{-(\gamma_1 z+\gamma_2 h)} + \gamma_1 a_1 t_{13}\mathrm{e}^{-(\gamma_1 z+\gamma_3 h)} + \gamma_2 a_2 t_{21}\mathrm{e}^{-(\gamma_2 z+\gamma_1 h)} + \gamma_2 a_2 t_{23}\mathrm{e}^{-(\gamma_2 z+\gamma_3 h)} \\ &+ t_{31}\mathrm{e}^{-(\gamma_3 z+\gamma_1 h)} + t_{32}\mathrm{e}^{-(\gamma_3 z+\gamma_2 h)}\big\} \end{aligned} \tag{A.25}$$

$$\begin{aligned}\overline{p}_1(\xi,z) = \overline{p}_{1\infty}(\xi,z) + \frac{1}{4\pi\mu a^2 S^2}\big\{ &t_{11}\mathrm{e}^{-\gamma_1(z+h)} + t_{22}\mathrm{e}^{-\gamma_2(z+h)} + t_{12}\mathrm{e}^{-(\gamma_1 z+\gamma_2 h)} + t_{13}\mathrm{e}^{-(\gamma_1 z+\gamma_3 h)} \\ &+ t_{21}\mathrm{e}^{-(\gamma_2 z+\gamma_1 h)} + t_{23}\mathrm{e}^{-(\gamma_2 z+\gamma_3 h)}\big\} \end{aligned} \tag{A.26}$$

in which

$$t_{i1} = [2\delta_1\xi^2(f_{i2} - f_{i1})] - \frac{\xi f_{i3} g_5}{\gamma_1} + \frac{\xi f_{i4}\delta_3 S^2}{\gamma_1} \quad (i = 1,4) \tag{A.27}$$

$$t_{i2} = [2\delta_2\xi^2(f_{i1} - f_{i2})] + \frac{\xi f_{i3} g_6}{\gamma_2} - \frac{\xi f_{i4}\delta_3 S^2}{\gamma_2} \quad (i = 1,4) \tag{A.28}$$

$$t_{i3} = [2(\gamma_3^2 f_{i1} - \xi^2 f_{i2})] + 2\xi\gamma_3 f_{i3} \quad (i = 1,4) \tag{A.29}$$

$$g_5 = 2\delta_1\gamma_1^2 - \lambda^*\delta_1 L_1^2 + \alpha S^2\delta_3 \tag{A.30}$$

$$g_6 = 2\delta_2\gamma_2^2 - \lambda^*\delta_2 L_2^2 + \alpha S^2\delta_3 \tag{A.31}$$

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